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## $F$ -spin purity of $2^+_{\text{I}}$ states in even–even nuclei

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A procedure is proposed by which upper limits to the  $F$ -spin purity of  $2^+_{\text{I}}$  states in even–even nuclei can be estimated by an analysis of experimental  $g$ -factors. The procedure, applied to 17 nuclei with  $46 \leq Z \leq 78$ , showed that  $F < F_{\text{max}}$  admixtures up to 10% are not uncommon. A systematic behavior with  $Z$  is observed. In the present approach, these admixtures account in part for the observed deviations of the boson  $g$ -factors  $g_{\pi}$  and  $g_{\nu}$  from their “bare” values of 1 and 0.

The purity of low-lying nuclear states with respect to valence proton–neutron symmetry, algebraically expressed in terms of  $F$ -spin, has been the subject of several investigations [1–8]. It is widely believed that low-lying states have nearly maximal  $F$ -spin,  $F_{\text{max}} = (N_p + N_n)/4$ , i.e., they are essentially symmetric with respect to valence protons ( $N_p$ ) and neutrons ( $N_n$ ). However, some studies [1,3–5,7] have suggested significant deviations from this symmetry. In fact, a certain amount of  $F$ -spin breaking is usually invoked to account for M1 transition rates, which vanish unless either  $F$ -spin symmetry is broken or higher order operators are used.  $F < F_{\text{max}}$  admixtures of a few percent have been estimated by Harter et al. [4] for  $^{168}\text{Er}$ , based on the M1 transition properties. A different approach was used by Ginocchio and Kuyucak [3], who used pion charge exchange data on  $^{165}\text{Ho}$  and estimated ground state band  $F$ -spin admixtures of 6%–45%. Comparisons of IBA-2 calculations with the experimental spectrum led Novoselski and Talmi [7] to suggest symmetry breaking as large as 18% for the  $g.s.$  band of  $^{178}\text{Hf}$ .

In this work we show that under certain assumptions it is possible to obtain estimates of the  $F$ -spin purity of  $2^+_{\text{I}}$  states in even–even nuclei from the experimental  $g$ -factors of these states. In the IBA-2, the

$g$ -factor of the  $2^+_{\text{I}}$  state with maximal  $F$ -spin is given by

$$g(2^+_{\text{I}}) = (g_{\pi}N_{\pi} + g_{\nu}N_{\nu}) / (N_{\pi} + N_{\nu}), \quad (1)$$

with  $N_{\pi}$  ( $N_{\nu}$ ) the numbers of valence proton (neutron) bosons, and  $g_{\pi}$ ,  $g_{\nu}$  the respective boson  $g$ -factors with “bare” values of 1, 0 respectively. Analysis of data has shown [9] that for many nuclei  $g_{\pi} \sim 0.7$ , and  $g_{\nu} \sim 0.0$ . These deviations from the “bare” values could arise from complicated single particle boson structure, although the deviations are larger than expected in most models. Another possible explanation, which we pursue here, is based on the fact that eq. (1) is valid only for pure  $F_{\text{max}}$  states. Therefore, the need to use  $g_{\pi}$ ,  $g_{\nu}$  values differing from their “bare” values in eq. (1) could mock up effects of  $F < F_{\text{max}}$  admixtures. We use this idea to estimate the symmetry breaking, but stress that other mechanisms [10,11] could explain  $g_{\pi}$ ,  $g_{\nu}$  values different from the “bare” values; hence our analysis can only provide upper limits for  $F$ -spin admixtures. (A possible exception occurs if microscopic calculations of the boson structure actually increase the boson  $g$ -values; then the actual  $F$ -spin admixtures could exceed the present estimates.)

A related idea, to use ratios of  $\gamma$  and  $g.s.$  band magnetic moments to study  $F$ -spin symmetry breaking

was recently proposed by Ginocchio et al [2]. While this method is quite sensitive to  $F$ -spin admixtures, it has the drawback that accurate data on  $\gamma$  band magnetic moments are rather sparse. On the other hand, many  $g$ -factors of  $2^+_{\pi}$  states are known to within 5%–10%, making the procedure proposed here more amenable to comparison with experiment. Our procedure is as follows. With bare boson  $g$ -factors, eq (1) is

$$g(2^+_{\pi}) = N_{\pi} / (N_{\pi} + N_{\nu}) \quad (2)$$

We ascribe deviations of experimental  $g(2^+_{\pi})$  values from eq (2) to  $F$ -spin admixtures. Using a modified version [12] of NPBOS, we performed IBA-2 calculations for 17 isotopes in the range  $Z=46$ –78 to fit these  $g(2^+_{\pi})$  values and other key observables and obtain upper limits on  $F$ -spin admixtures by projecting the resultant wave functions onto states with good  $F$ -spin ( $F_{\max}, F_{\max}-1, \dots$ ).

The starting point was to use for each isotope previously established [13–19] Hamiltonians. In general, two types were used

$$H_1 = n_{\pi}(\epsilon_d + \epsilon_{d\pi}) + n_{\nu}(\epsilon_d + \epsilon_{d\nu}) + \kappa Q_{\pi} \cdot Q_{\nu} + \lambda M, \quad (3)$$

$$H_2 = n_{\pi}(\epsilon_d + \epsilon_{d\pi}) + n_{\nu}(\epsilon_d + \epsilon_{d\nu}) + \kappa(Q_{\pi} + Q_{\nu})^2 + \lambda M, \quad (4)$$

where

$$Q_{\rho} = (d_{\rho}^{\dagger} + s_{\rho})^2 + \chi_{\rho}(d_{\rho}^{\dagger} d_{\rho})^2, \quad \rho = \pi, \nu,$$

and  $M$  is the Majorana operator defined in ref [11]. In some cases, an additional boson–boson interaction term was used in either  $H_1$  or  $H_2$ . The isotopes studied were  $^{110}\text{Pd}$ ,  $^{152}\text{Sm}$ ,  $^{154}\text{Gd}$ ,  $^{168}\text{Er}$ ,  $^{184}\text{W}$ , using  $H_2$  and parameters from refs [13,17],  $^{130}\text{Ba}$ ,  $^{186}\text{Os}$ , using both  $H_1$  and  $H_2$ , and parameters from refs [14,18,19],  $^{146}\text{Ce}$ ,  $^{150}\text{Nd}$ ,  $^{192}\text{Pt}$ ,  $^{194}\text{Pt}$ ,  $^{196}\text{Pt}$  with  $H_1$  and parameters from refs [16,18]. We also made calculations for  $^{170}\text{Yb}$  and  $^{180}\text{Hf}$  with Hamiltonian  $H_2$  and parameters from ref [17], since these nuclei are similar to those studied in ref [17]. The Majorana parameter  $\lambda$  was taken as [15],  $\lambda = 3.4\delta / (N_{\pi}N_{\nu})^{1/2}$ , where  $\delta$  is the mass deformation parameter. As we will see, the results are not very sensitive to the particular choice of the Hamiltonian or its parameters.

We use the parameters  $\epsilon_{d\pi}$  and  $\epsilon_{d\nu}$  as a convenient way to incorporate separate neutron and proton boson energies, and fit the  $g$ -factors. Though they are

traditionally taken as  $\epsilon_{d\pi} = \epsilon_{d\nu} = 0$ , calculated  $g(2^+_{\pi})$  values are actually quite sensitive to  $\epsilon_{d\pi}$ ,  $\epsilon_{d\nu}$ . An alternative approach is to use  $\chi_{\pi} \neq \chi_{\nu}$ . These parameters, related to the  $p$ ,  $n$  deformations, were indeed found [2] to affect  $g$  s and  $\gamma$  band magnetic moments. However, test calculations showed that any reasonable choice of  $\chi_{\pi}$ ,  $\chi_{\nu}$  did not suffice to fit the  $g$ -factors. Moreover,  $\chi_{\pi}$ ,  $\chi_{\nu}$  considerably affect M1 transition rates and hence cannot be freely varied.

Of course, the introduction of  $\epsilon_{d\pi} \neq \epsilon_{d\nu} \neq 0$  causes  $F < F_{\max}$  admixtures to appear in the low-lying excited states, with effects on calculated energy levels and transition rates. To minimize the change in calculated energy levels, we used the condition

$$N_{\pi}\epsilon_{d\pi} + N_{\nu}\epsilon_{d\nu} = 0 \quad (5)$$

With this condition, only one of  $\epsilon_{d\pi}$ ,  $\epsilon_{d\nu}$  is free to vary, for convenience, we therefore define  $\epsilon_{\nu} \equiv \epsilon_{d\nu} - \epsilon_{d\pi}$ , and use  $\epsilon_{\nu}$  as a free parameter. A constraint was imposed, namely, that  $\epsilon_{\rho} \equiv \epsilon_d + \epsilon_{d\rho} \geq 0$ . When our procedure led to  $\epsilon_{\rho} < 0$ ,  $\epsilon_d$  was modified to satisfy this constraint, and  $\kappa$ ,  $\chi_{\rho}$  were slightly changed to fit again the other observables. Fig. 1 illustrates the sensitivity of  $g(2^+_{\pi})$  to different Hamiltonian parameter, and to the  $F$ -spin admixtures. For  $^{154}\text{Gd}$  the intersection with the experimental value requires quite large  $\epsilon_{\nu}$  values. We note that  $\epsilon_{\nu}$  is used here as a “generic” parameter to generate the necessary  $F$ -spin breaking. Fig. 1a shows that, depending on the structure of the Hamiltonian used (in this case, different values of  $\lambda$ ), quite different  $\epsilon_{\nu}$  values are required to reproduce the measured  $g$ -factor. However, fig. 1b shows that very similar percentages of  $F < F_{\max}$  admixtures in the  $2^+_{\pi}$  state resulted. In most cases, rather large, negative values of  $\epsilon_{\nu}$  were needed to fit the data. Qualitatively, negative  $\epsilon_{\nu}$  values, implying  $\epsilon_{d\pi} > \epsilon_{d\nu}$ , are microscopically reasonable since proton  $2^+$  excitations generally lie higher than neutron  $2^+$  excitations. The Hamiltonian dependence of the deduced admixtures is illustrated for hamiltonians  $H_1$  and  $H_2$  in fig. 1c for  $^{132}\text{Ba}$ . The intersection of the calculated curves with the experimental value gives similar admixtures. The essential point is that, whatever Hamiltonian is used to fit the  $g$ -factor data, similar  $F$ -spin admixtures and hence upper limits for the  $F$ -spin purity are obtained.

Before presenting the results, one more issue remains to be discussed, namely, the effect of  $\epsilon_{\nu} \neq 0$  on other calculated observables, e.g., energy levels and

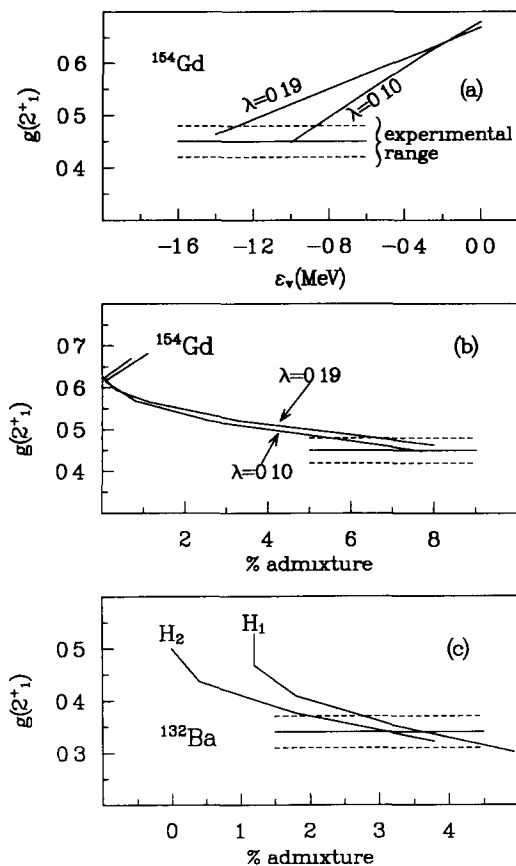


Fig. 1 (a) Sensitivity of calculated  $g(2^+_1)$  to parameter  $\epsilon_v$  for  $^{154}\text{Gd}$ , (b) calculated  $g(2^+_1)$  versus percent admixture of  $F < F_{\max}$  states into the  $2^+_1$  state for two values of the Majorana parameter  $\lambda$ , for  $^{154}\text{Gd}$ , (c) calculated  $g(2^+_1)$  versus percent admixture for two different Hamiltonians  $H_1, H_2$ , for  $^{132}\text{Ba}$ . In all three parts the experimental  $g(2^+_1)$  value, with its uncertainty, is indicated by horizontal bars.

transition rates. Energies are not very sensitive to  $\epsilon_v$ , and can always be further improved by adjustments of  $\epsilon_d$  and  $\kappa$ . Transition rates (especially M1 rates) are more strongly affected by  $\epsilon_v$ , but fine-tuning of  $\chi_\pi, \chi_\nu$  usually yields reasonable agreement with experiment or with calculations in which  $\epsilon_v = 0.0$ . Magnetic moments are not sensitive to the detailed values of most IBA parameters, because eq. (1) is valid for any system of bosons, as long as  $F$ -spin is conserved. Table 1 compares several M1 reduced matrix elements and  $g(2^+_1)$  for  $^{154}\text{Gd}$ , calculated using  $\epsilon_v = 0.0$  and  $\epsilon_v = -1.0$  MeV, and shows good agreement of both calculations with experiment. Both calculations used the parameters of ref. [17], except for some changes in  $\chi_\pi, \chi_\nu$ . The advantage of using  $\epsilon_v \neq 0.0$  is obviously that it enables a reasonably good description of both M1 rates and the magnetic moment.

We used the same procedure for all 17 isotopes studied. The  $g(2^+_1)$  data were taken from ref. [20]. The results are given in fig. 2a. They show limits on  $F$ -spin admixtures varying up to  $\sim 10\%$  and clear systematics: the % admixtures rise considerably around  $Z=64$ , decrease towards  $Z=74$ , and increase again for  $Z=76, 78$ . This pattern is certainly not unexpected since the deviation of experimental data from the fully symmetric prediction (eq. (2)) is much larger around  $Z=64$  than in other regions (fig. 2b). Therefore, if one attributes this deviation to  $F$ -spin admixture, results with the qualitative shape of fig. 2a would be expected almost regardless of the model used. This validates our procedure and helps account for the stability of the  $F$ -spin admixtures using different Hamiltonians. In fact numerous test calculations

Table 1

Calculated and experimental values of M1 reduced matrix elements (in  $\mu_N$ ) and  $g(2^+_1)$  for several transitions in  $^{154}\text{Gd}$

$J_i$	$J_f$	$\epsilon_v$ (MeV)		Experiment <sup>a)</sup>
		0.0 <sup>a)</sup>	-1.0 <sup>b)</sup>	
$2\gamma$	$2g$	0.045	0.042	0.044(3)
$3\gamma$	$2g$	0.053	0.054	0.066(4)
$3\gamma$	$4g$	0.075	0.080	0.072(3)
$4\gamma$	$4g$	0.120	0.072	0.16(2)
$3\gamma$	$2\gamma$	0.152	0.110	0.20(5)
$g(2^+_1)$		0.670	0.517	0.45(3)

<sup>a)</sup> From ref. [17]

<sup>b)</sup> Present work,  $\chi_\pi = -1.2, \chi_\nu = -0.7$  (instead of  $-1.6, -0.2$  in ref. [17])

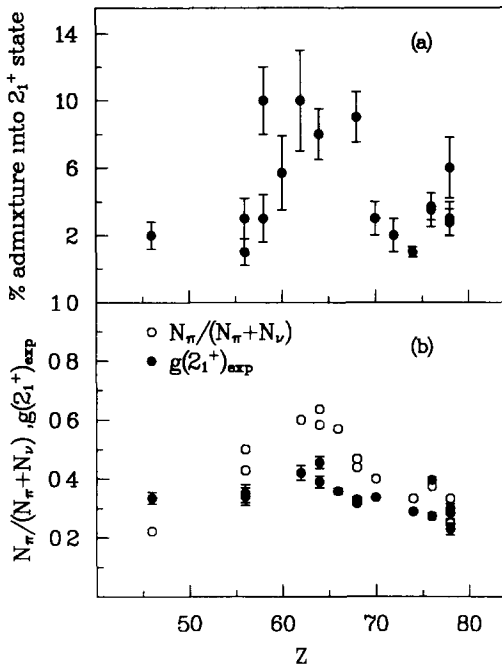


Fig. 2 (a) Upper limits of percent admixtures of  $F < F_{\text{max}}$  states in the  $2_1^+$  state for the isotopes studied, (b) experimental  $g$ -factors and the ratio  $N_\pi/(N_\pi+N_\nu)$  (eq. (2)) for the same isotopes

(e.g.  $^{132}\text{Ba}$ , see above) with a variety of Hamiltonians confirm this fact. Moreover, for Sm, Gd, Er, and W we used only  $H_2$  with the parameters of ref. [17], and yet obtained quite different admixtures for the respective isotopes. It therefore seems quite plausible that within the assumptions of the present study the structure in fig. 2a is real, and not a numerical artifact. It is interesting to note that a very similar behavior has been observed [21] for the ratio  $e_\nu/e_\pi$  of the neutron, proton effective charges. The relationship between effective charges, the proton and neutron deformations, and  $F$ -spin admixtures has been recently discussed by Ginocchio and Kuyucak [3]. They pointed out that in some cases, when the ratio  $e_\nu/e_\pi$  is close to unity, quite large  $F$ -spin admixtures may be required in the  $g$  s band in order to account for differences in proton and neutron deformations. We also note that our approach is closely related to one in terms of intrinsic states for the IBA [2,3].  $F$ -spin admixtures are related to differences in  $p$  and  $n$  deformations, and eq. (2) can be written in terms of these deformations, and subsequently  $F$ -spin ampli-

tudes can be extracted. This approach yields admixtures with slightly smaller magnitudes and with similar systematics as those in fig. 2, supporting our point that the admixtures obtained here are quite independent of the calculational details.

In conclusion, we have shown that upper limits for  $F$ -spin admixtures can be estimated by using experimental nuclear  $g(2_1^+)$ -factors. Results suggesting  $F < F_{\text{max}}$  admixtures of up to about 10% are not uncommon. In some cases, these admixtures are larger than those obtained when only the M1 rates are considered. For example, in  $^{168}\text{Er}$  we find an upper limit of about 9%, while Harter et al. [4] reported an upper limit of only 4% for the same nucleus, from a fit to M1 transitions. However, unlike magnetic moments, M1 transition rates are rather sensitive to the precise form of the Hamiltonian, and thus an alternate method to extract  $F$ -spin admixtures is useful. A correlation with the effective charges in the respective isotopes is observed. This points to a relation between effective charges and  $F$ -spin symmetry breaking that is not completely understood but warrants further investigation. The viewpoint of the present approach is that at least part of the empirical deviations of the boson  $g$ -factors  $g_\pi$  and  $g_\nu$  from their "bare" values can be interpreted in terms of  $F$ -spin admixtures. This approach is complementary to the usual one, which assumes eq. (1) to be correct (thus ignoring any  $F$ -spin impurity) and varies the boson  $g$ -factors to reproduce measured magnetic moments.

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## References

- [1] P.O. Lipas, P. von Brentano and A. Gelberg, Rep. Prog. Phys. 53 (1990) 1355.
- [2] J.N. Ginocchio, W. Frank and P. von Brentano, Nucl. Phys. A 541 (1992) 211.
- [3] J.N. Ginocchio and S. Kuyucak, Phys. Rev. C 45 (1992) 867.
- [4] H. Harter et al., Phys. Lett. B 188 (1987) 295.

- [5] J N Ginocchio, A Leviatan and M W Kirson, in Capture gamma-ray spectroscopy, Proc 7th Intern Symp on Capture gamma-rays spectroscopy and related topics, ed R W Hoff, AIP Conf proc No 238 (AIP, New York, 1991)
- [6] W Gelletly et al, Phys Lett B 191 (1987) 240
- [7] A Novoselski and I Talmi, Phys Lett B 160 (1985) 13
- [8] A E L Dieperink, O Scholten and D D Warner, Nucl Phys A 469 (1987) 173
- [9] A Wolf, R F Casten and D D Warner, Phys Lett B 190 (1987) 19
- [10] P O Lipas et al, Nucl Phys A 509 (1990) 509
- [11] M Sambataro et al, Nucl Phys A 423 (1984) 333
- [12] H Harter, unpublished
- [13] U Hartmann et al, Nucl Phys A 465 (1987) 25
- [14] G Puddu, O Scholten and T Otsuka, Nucl Phys A 348 (1980) 109
- [15] H Harter et al, Phys Lett B 205 (1988) 174
- [16] R L Gill et al, Phys Lett B 118 (1982) 251
- [17] P Van Isacker et al, Nucl Phys A 476 (1988) 301
- [18] R Bijker, A E L Dieperink and O Scholten, Nucl Phys A 344 (1980) 207
- [19] W Krips et al, Nucl Phys A 529 (1991) 485
- [20] P Raghavan, At Data Nucl Data Tables 42 (1989) 189
- [21] R F Casten and A Wolf, Phys Lett B 280 (1992) 1